CSL model checking

Quantitative Logics

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David N. Jansen

Recapitulation: CTMC

- A continuous-time Markov chain consists of:
  - S — finite set of states (often $S = \{1, 2, \ldots n\}$)
  - $\mathbf{R}: S \times S \rightarrow \mathbb{R}_{\geq 0}$ — transition rate matrix
  - π_0: S → [0,1] — initial state distribution (sometimes)
  - $L: S \rightarrow AP$ — labelling with atomic propositions

Recapitulation: CSL

- state formulas ϕ, ψ
  - a — atomic proposition
  - $\neg\phi$ — negation
  - $\phi \vee \psi$ — disjunction
  - $\mathbf{P}_{\leq p}(\Pi)$, $\mathbf{P}_{\geq p}(\Pi)$ — probabilistic operator
  - $\mathbf{S}_{\leq p}(\phi)$, $\mathbf{S}_{\geq p}(\phi)$ — steady-state operator
- path formulas Π
  - $X^{I}\,\phi$ — next state
  - $\phi\ U^{I}\,\psi$ — until

with time bound:
an interval $I \subseteq \mathbb{R}_{\geq 0}$

CSL Model checking

- Assume given a CTMC (S, $\mathbf{R}$, L) and a formula ϕ
- Find, for each state formula ψ that is in ϕ, the set of states that satisfies it, $Sat(\psi)$
- Find, for each path formula Π that is in ϕ, the set of paths that satisfies it, $Sat(\Pi)$

Simple formulas

- Atomic proposition:
  $Sat(a) = \{ s \in S \mid a \in L(s) \}$
- Negation:
  $Sat(\neg\phi) = S \setminus Sat(\phi)$
- Disjunction:
  $Sat(\phi \vee \psi) = Sat(\phi) \cup Sat(\psi)$

Next formulas

- $Sat(\mathbf{P}_{\leq p}(\mathrm{X}^{I}\,\phi)) = \{\, s \in S \mid Prob_s(Sat(\mathrm{X}^{I}\,\phi)) \leq p \,\}$
- Do not actually calculate this satisfaction set!

- This is similar to PCTL next formula & probability to take the transition in interval I.

Until formulas

- $Sat(\mathbf{P}_{\leq p}(\phi \ \mathrm{U}^{I} \ \psi))$
  $= \{ s \in S \mid Prob_s(Sat(\phi \ \mathrm{U}^{I} \ \psi)) \leq p \}$

- There is not a fixed number of transitions in this interval.
- Use Fox–Glynn sum, combined with time-bounded until model checking

- we are going to make this slide together

Steady-state formulas

- $Sat(\mathbf{S}_{\leq p}(\phi)) =$
  $\{ s \in S \mid \pi_s \leq p$ for the equation $\pi Q = 0 \}$