Quantitative Logics
Exercises 4

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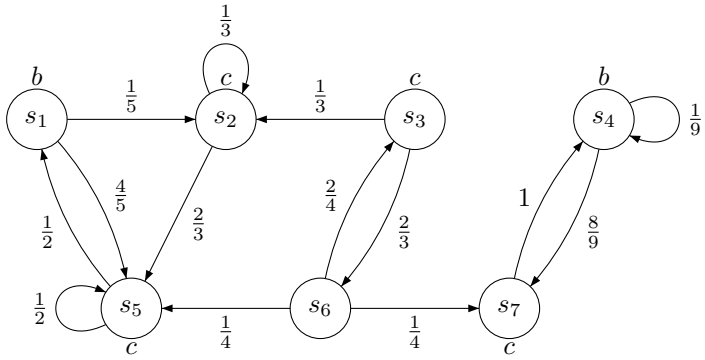
1. There is a relation between PCTL and CTL: E$\Diamond\,\varphi$ correspondends to $P_{>0}(\mathit{true}\ U\ \varphi)$. To what PCTL formula does A$\Diamond\,\varphi$ correspond? Are these correspondences always exact?

2. Construct the (in)equation system to verify $\mathcal{P}_{<0.5}(c\,U\,b)$ on the following DTMC and show how to deduce the satisfaction set from its solution. Atomic propositions that hold in a state are noted close to that state.



3. We will prove a part of Theorem 10.15 (from the lecture). We define the operator $\mathcal{Y} : [0,1]^S \to [0,1]^S$ as in the lecture:

$$\begin{aligned}
&\mathcal{Y}(x)(s) = 1 && \text{if } s \in \mathit{Sat}(\mathit{win}) \\
&\mathcal{Y}(x)(s) = 0 && \text{if } s \in \mathit{Sat}(\mathit{lose}) \\
&\mathcal{Y}(x)(s) = \sum_{t \in S} P(s,t)x(t) && \text{otherwise}
\end{aligned}$$

Let $x^{(-1)}$ be the vector containing all zeroes. For $n \geq 0$, let $x^{(n)} := \mathcal{Y}(x^{(n-1)})$.

Prove that $x^{(n)}(s)$ is the probability that the DTMC takes a path in $\mathit{Sat}(\neg \mathit{lose}\ U^{\leq n}\ \mathit{win})$, if it starts in s.