Quantitative Logics
Exercises 1

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1. I mentioned several possible sources of "randomness". Which sources do the following authors speak of?

   - Charles Darwin: **On the origin of species.** London: Murray, 1859, p. 131.
     "I have hitherto sometimes spoken as if the variations [] had been due to chance. This, of course, is a wholly incorrect expression, but it serves to acknowledge plainly our ignorance of the cause of each particular variation. Some authors believe it to be as much the function of the reproductive system to produce individual differences [] as to make the child like its parents. But the much greater variability [] under domestication [] than under nature, leads me to believe that deviations [] are [] due to [] the conditions of life, to which the parents and their more remote ancestors have been exposed."
   - Elliott Sober: **Evolution without naturalism.** In: Jonathan L. Kvanvig (ed.): *Oxford studies in philosophy of religion,* vol. 3. Oxford: Oxford Univ. Pr., 2011.
     "There is no physical mechanism (either inside organisms or outside of them) that [] causes [] mutations to occur."
   - De Laplace: **Essai philosophique sur les probabilités.** Paris: Bachelier, 1840, pp. 3–4:
     « Nous devons donc envisager l'état présent de l'univers, comme l'effet de son état antérieur, et comme la cause de celui qui va suivre. Une intelligence qui [] connaîtrait toutes les forces dont la nature est animée, et la situation respective des êtres qui la composent [] embrasserait dans la même formule les mouvemen[t]s des plus grands corps de l'univers et ceux du plus léger atome : rien ne serait incertain pour elle, et l'avenir comme le passé, serait présent à ses yeux. »[1]
   - Alvin Plantinga: **Where the conflict really lies.** Oxford: Oxford Univ. Pr., 2011, pp. 39–40.
     "The truth of the theory of natural selection [] doesn't [] show that all of life has come to be by way of *unguided* natural selection, or even that it is biologically possible that it has come to be that way."

2. Can you simplify the Craps game Markov chain? By "simplify" I mean find a Markov chain with fewer states that has the same probability to win.

[1] Laplace: **A philosophical essay on probabilities.** tr. F. W. Truscott and E. L. Emory. New York: Dover, 1812, p. 4: "We ought then to regard the present state of the universe as the effect of its previous state and as the cause of the one which is to follow. Given [] a mind which could comprehend all the forces by which nature is animated and the respective situation of the beings that compose it [], it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes."

3. Translate the following sentences to CTL formulas. Define sensible atomic propositions wherever necessary.

   (a) I will never be drunk again.

   (b) It may happen that I will get drunk one day.

   (c) It will definitely happen that I will get drunk one day.

   (d) There may come a day after which I will never be drunk again.

   (e) Change the above formulas to take into account my possible death; e. g. change (3c) to "It will definitely happen ... except if I die before that."

4. During the lesson I said that the following two definitions of the Borel-σ-algebra are equivalent:

   - The smallest σ-algebra over $\mathbb{R}$ that contains all intervals $[r, s)$, for $r, s \in \mathbb{R}$.
   - The smallest σ-algebra over $\mathbb{R}$ that contains all intervals $[r, s]$, for $r, s \in \mathbb{R}$.

   Prove the equivalence of the two definitions.

   (Hint: Try a concrete example first, e. g. prove that $[2, 3]$ is in the first σ-algebra.)