Quantitative Logics
CTMC exercises

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1. (From last year's exam:) An aeroplane interior designer is considering how many toilets to place in the plane. In the first class cabin, there is one toilet for the 12 seats. Assume that every passenger visits the toilet once in two hours and stays there for five minutes, on average. The designer finds it acceptable that there is a queue of two (or more) waiting passengers in 10 % of the five-hour flights.

   Produce a CTMC model to verify this property and provide a CSL formula. Explain by which (in)equation system you would verify the formula.

2. Normally, the set of paths that take a transition at an exact time point has probability zero. Therefore, in most situations, the set of paths $\{\sigma \mid \sigma \text{ satisfies } a\, U^{[1,1]}\, b\}$ has probability zero. However, in some special cases, the probability is not zero. In which special cases?

3. I gave a very sloppy proof of the relation between the rate matrix $\mathbf{R}$ and the infinitesimal generator matrix $\mathbf{Q} = \mathbf{P}'(0) = \lim_{dt\to 0}[\mathbf{P}(dt) - \mathbf{P}(0)]/dt$ of a CTMC. Make this relation more exact, using the equation $\mathbf{P}(dt) = \sum_{i=0}^{\infty} \Pr(i \text{ transitions within time } dt) \cdot P^i$, where P is the single-step transition matrix of the DTMC constructed by uniformisation. You will see that as dt goes to zero, the probability to take more than one transition can be neglected. Hint: Assume that $\mathbf{R}$ is already uniform with exit rate E; then $P = \frac{1}{E}\mathbf{R}$.